CHAPTER 12

Introduction to the Laplace Transform

TABLE 12.1 An Abbreviated List of Laplace Transform Pairs

Type	$f(t)\ (t > 0-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

TABLE 12.2 An Abbreviated List of Operational Transforms

Operation	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
nth derivative (time)	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
nth derivative (s)	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$

12.7 Inverse Transforms

Partial Fraction Expansion:

CHAPTER 13

The Laplace Transform in Circuit Analysis

13.1 Circuit Elements in the *s* Domain

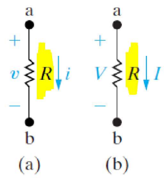


Figure 13.1 ▲ The resistance element. (a) Time domain. (b) Frequency domain.

$v = Ri$ Time Domain

$\downarrow \mathcal{L}$

$V = RI$ s-Domain

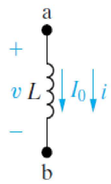
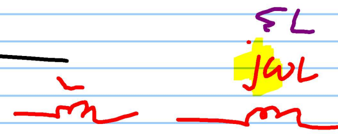


Figure 13.2 ▲ An inductor of L henrys carrying an initial current of I_0 amperes.

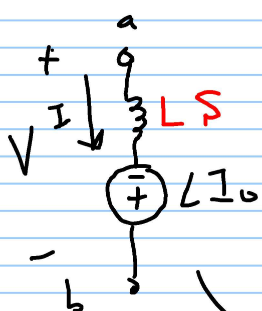


$\mathcal{L}\left(v = L \frac{d}{dt} i\right)$

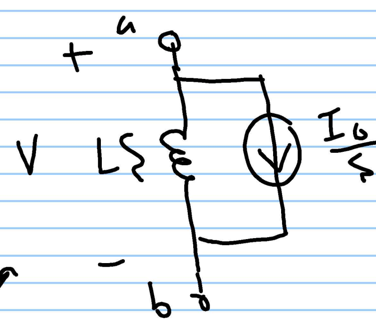
$\boxed{s = j\omega}$

$V = L\left[sI - i(0^-)\right]$

$(V = LsI - LI_0)$



Source Transformation



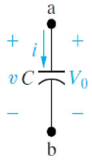
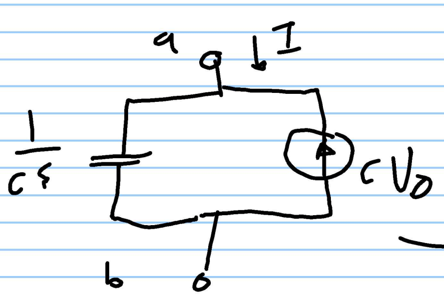


Figure 13.6 ▲ A capacitor of C farads initially charged to V_0 volts.

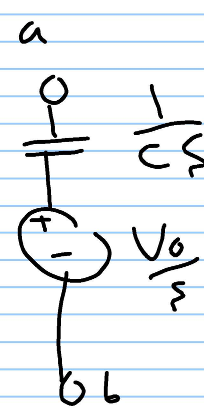
$\mathcal{L}\left(i = c \frac{d}{dt} v\right)$

$I = C\left[sV - V(0^-)\right]$

$I = CsV - CV_0$



Source Transf



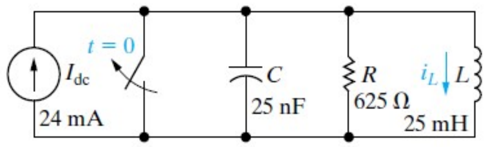
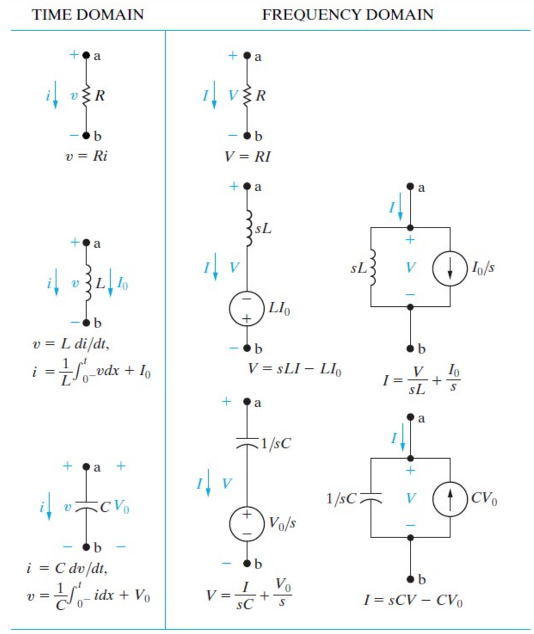


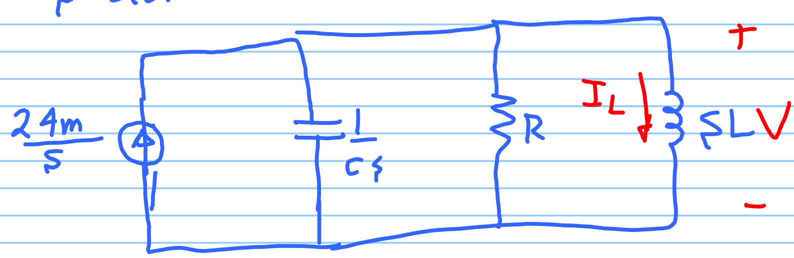
Figure 13.13 ▲ The step response of a parallel *RLC* circuit.

TABLE 13.1 Summary of the s-Domain Equivalent Circuits



find $i_L(t)$, V_0 & I_0 = zero

s-domain



$$\frac{24m}{s} = CsV + \frac{V}{R} + \frac{V}{sL}$$

$$V = \frac{24m/C}{s^2 + \frac{1}{RC}s + 1/LC}, \quad I_L = \frac{V}{sL}$$

$$\text{so } I_L = \frac{24m/LC}{s\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)}$$

$$= \frac{384\times10^5}{s(s^2 + 64000s + 16\times10^8)} + \left(\frac{64000}{2}\right)^2 - \left(\frac{64000}{2}\right)^2$$

$$= \frac{384\times10^5}{s\left[\left(s^2 + 64000s + 1.024\times10^9\right) + 576\times10^6\right]}$$

$$= \frac{384\times10^5}{s\left[(s+32000)^2 + 576\times10^6\right]}$$

$$= \frac{K_1}{s} + \frac{K_2 s + K_3}{(s+32000)^2 + 576\times10^6}$$

$K_1 = 24\times10^{-3}, \quad K_2 = -24\times10^{-3}, \quad K_3 = -1536$

$$\therefore I_L = \frac{24\times10^{-3}}{s} - 24\times10^{-3}\,\frac{s+64000}{(s+32000)^2+576\times10^6}$$

$\cos\omega t \leftrightarrow \dfrac{s}{s^2+\omega^2}$

$\sin\omega t \leftrightarrow \dfrac{\omega}{s^2+\omega^2}$

$$= \frac{24\times10^{-3}}{s} - 24\times10^{-3}\,\frac{(s+32000)+32000}{(s+32000)^2+576\times10^6}$$

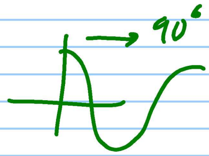
$$= \frac{24\times10^{-3}}{s} - 24\times10^{-3}\left[\frac{(s+32000)}{(s+32000)^2+576\times10^6} + \frac{32000}{(s+32000)^2+576\times10^6}\right]$$

$$= \frac{24\times10^{-3}}{s} - 24\times10^{-3}\left[\frac{s+32000}{(s+32000)^2+(24000)^2} + \frac{4}{3}\,\frac{24000}{(s+32000)^2+(24000)^2}\right]$$

$\mathcal{L}^{-1}$

$$i_L(t) = 24m - 24m\left(e^{-32000t}\cos 24000t + \frac{4}{3}\sin 24000t\, e^{-32000t}\right)$$

$$= 24m - 24m\,e^{-32000t}\left(\cos 24000t + \frac{4}{3}\sin 24000t\right)$$



$1\angle 0 + \frac{4}{3}\angle -90$

$1 - j\frac{4}{3}$

$\frac{5}{3}\angle -53.13$

$$\therefore i_L(t) = 24m - 40m\,e^{-32000t}\cos(24000t - 53.13)\ \text{A} \ \checkmark$$

$$\text{OR } i_L(t) = 24m + 40m\,e^{-32000t}\cos(24000t + 126.87)\ \text{A} \ \checkmark$$

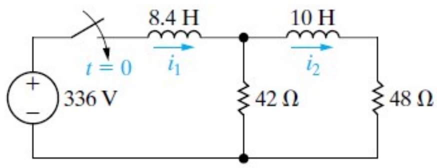


Figure 13.15 ▲ A multiple-mesh *RL* circuit.

Figure P13.86